

Recent progress in calculating B_K using staggered fermions

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SWME Collaboration

Staggered ε'/ε Project

1998 — Present

SWME Collaboration

- Seoul National University (SNU):
Prof. Weonjong Lee
Jon Bailey and Nigel Cundy (Research Assistant Prof's)
10 graduate students.
- Brookhaven National Laboratory (BNL):
Dr. Chulwoo Jung.
Dr. Hyung-Jin Kim (Postdoc).
- University of Washington, Seattle (UW):
Prof. Stephen R. Sharpe.
- KISTI: Dr. Taegil Bae (Postdoc).
- University of Arizona: Dr. Jongjeong Kim (Postdoc).

Indirect CP Violation and B_K

ε and \hat{B}_K

- $\varepsilon = (2.228 \pm 0.011) \times 10^{-3} \times e^{i\pi/4}$ in experiment.

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- Relation between ε and \hat{B}_K in standard model.

$$\varepsilon = \exp(i\phi_\varepsilon) \sqrt{2} \sin(\phi_\varepsilon) C_\varepsilon \operatorname{Im} \lambda_t X \hat{B}_K + \xi$$

$$X = \operatorname{Re} \lambda_c [\eta_1 S_0(x_c) - \eta_3 S_3(x_c, x_t)] - \operatorname{Re} \lambda_t \eta_2 S_0(x_t)$$

$$\lambda_i = V_{is}^* V_{id}, \quad x_i = m_i^2 / M_W^2, \quad C_\varepsilon = \frac{G_F^2 F_K^2 m_K M_W^2}{6\sqrt{2}\pi^2 \Delta M_K}$$

$$\xi = \exp(i\phi_\varepsilon) \sin(\phi_\varepsilon) \frac{\operatorname{Im} A_0}{\operatorname{Re} A_0}$$

ε and \hat{B}_K

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- Relation between ε and \hat{B}_K in standard model.

$$\begin{aligned}\varepsilon &= \exp(i\phi_\varepsilon) \sqrt{2} \sin(\phi_\varepsilon) C_\varepsilon \operatorname{Im} \lambda_t X \hat{B}_K + \xi \\ X &= \operatorname{Re} \lambda_c [\eta_1 S_0(x_c) - \eta_3 S_3(x_c, x_t)] - \operatorname{Re} \lambda_t \eta_2 S_0(x_t) \\ \lambda_i &= V_{is}^* V_{id}, \quad x_i = m_i^2/M_W^2, \quad C_\varepsilon = \frac{G_F^2 F_K^2 m_K M_W^2}{6\sqrt{2}\pi^2 \Delta M_K} \\ \xi &= \exp(i\phi_\varepsilon) \sin(\phi_\varepsilon) \frac{\operatorname{Im} A_0}{\operatorname{Re} A_0}\end{aligned}$$

- Definition of B_K in standard model.

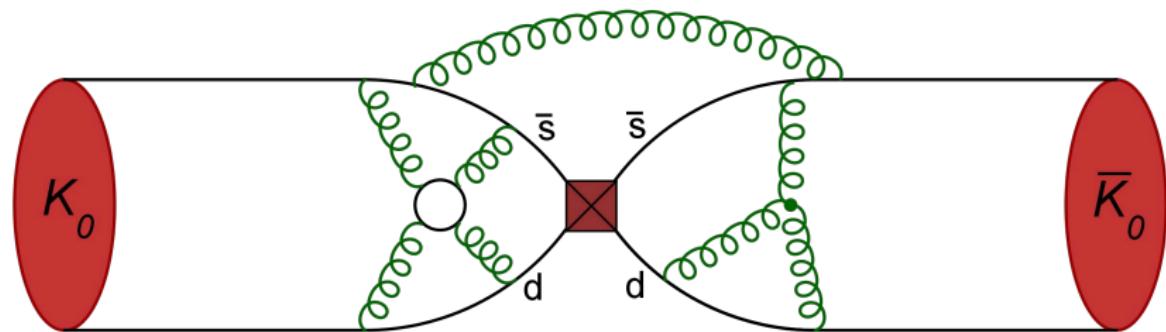
$$\begin{aligned}B_K &= \frac{\langle \bar{K}_0 | [\bar{s} \gamma_\mu (1 - \gamma_5) d] [\bar{s} \gamma_\mu (1 - \gamma_5) d] | K_0 \rangle}{\frac{8}{3} \langle \bar{K}_0 | \bar{s} \gamma_\mu \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_\mu \gamma_5 d | K_0 \rangle} \\ \hat{B}_K &= C(\mu) B_K(\mu), \quad C(\mu) = \alpha_s(\mu)^{-\frac{\gamma_0}{2b_0}} [1 + \alpha_s(\mu) J_3]\end{aligned}$$

B_K on the lattice

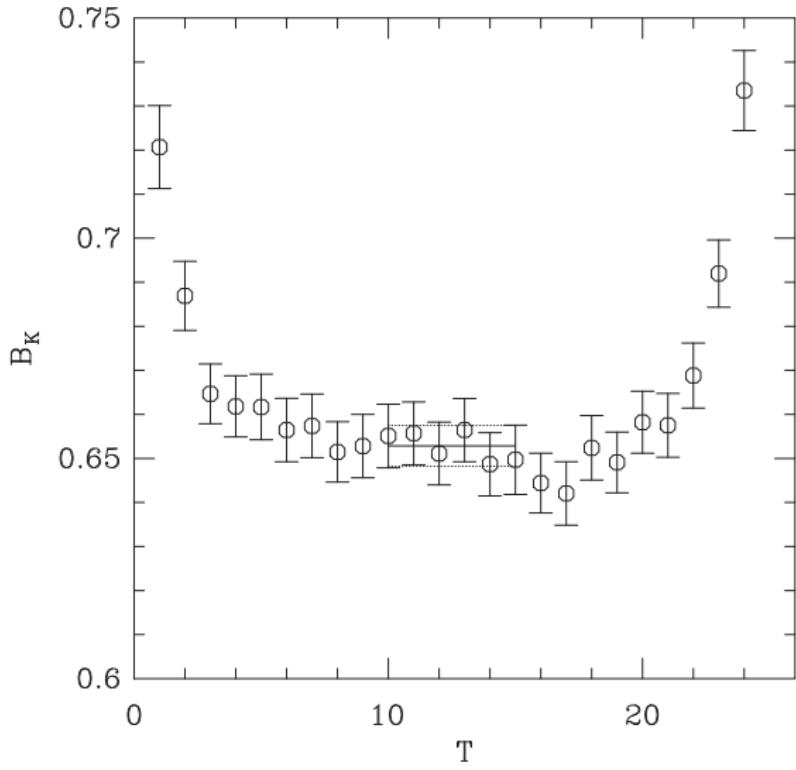
B_K definition in standard model

$$\begin{aligned}
 B_K &= \frac{\langle \bar{K}_0 | [\bar{s} \gamma_\mu (1 - \gamma_5) d] [\bar{s} \gamma_\mu (1 - \gamma_5) d] | K_0 \rangle}{\frac{8}{3} \langle \bar{K}_0 | \bar{s} \gamma_\mu \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_\mu \gamma_5 d | K_0 \rangle} \\
 \hat{B}_K &= C(\mu) B_K(\mu), \\
 C(\mu) &= \alpha_s(\mu)^{-\frac{\gamma_0}{2b_0}} [1 + \alpha_s(\mu) J_3]
 \end{aligned}$$

What do we calculate on the lattice?



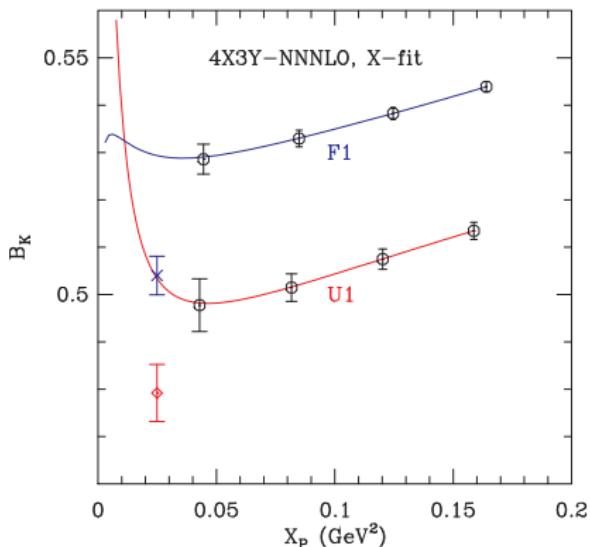
Data Analysis for B_K

Data for B_K with $am_d = am_s = 0.025$ ($20^3 \times 64$)

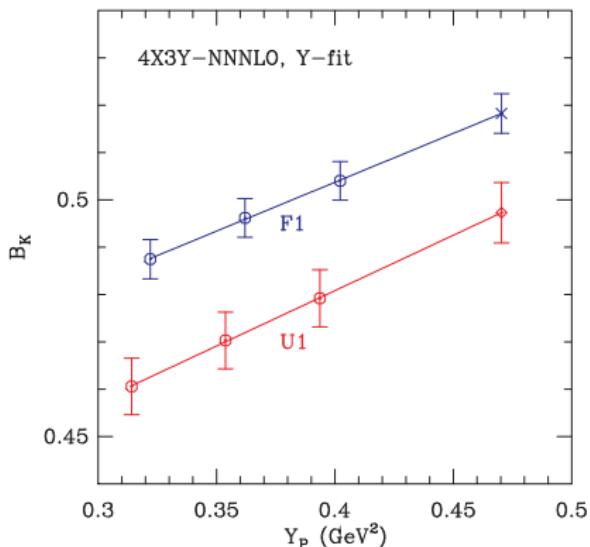
B_K in N_f = 2 + 1 QCD

a (fm)	am _I /am _s	geometry	ens × meas	round	production
0.12	0.03/0.05	20 ³ × 64	564 × 9	2nd	done (SNU)
0.12	0.02/0.05	20 ³ × 64	486 × 9	2nd	done (SNU)
0.12	0.01/0.05	20 ³ × 64	671 × 9	2nd	done (SNU)
0.12	0.01/0.05	28 ³ × 64	274 × 8	2nd	done (BNL)
0.12	0.007/0.05	20 ³ × 64	651 × 10	2nd	done (SNU)
0.12	0.005/0.05	24 ³ × 64	509 × 9	2nd	done (SNU)
0.09 (F1)	0.0062/0.031	28 ³ × 96	995 × 9	2nd	done (SNU)
0.09	0.0031/0.031	40 ³ × 96	805 × 1	2nd	SNU(*)
0.06	0.0036/0.018	48 ³ × 144	744 × 2	2nd	SNU/JLAB(
0.06	0.0025/0.018	56 ³ × 144	700 × 1	1st	KISTI(*)
0.06	0.0018/0.018	64 ³ × 144	700 × —	—	—
0.045 (U1)	0.0030/0.015	64 ³ × 192	705 × 1	2nd	SNU(*)

SU(2) SChPT Fitting for B_K



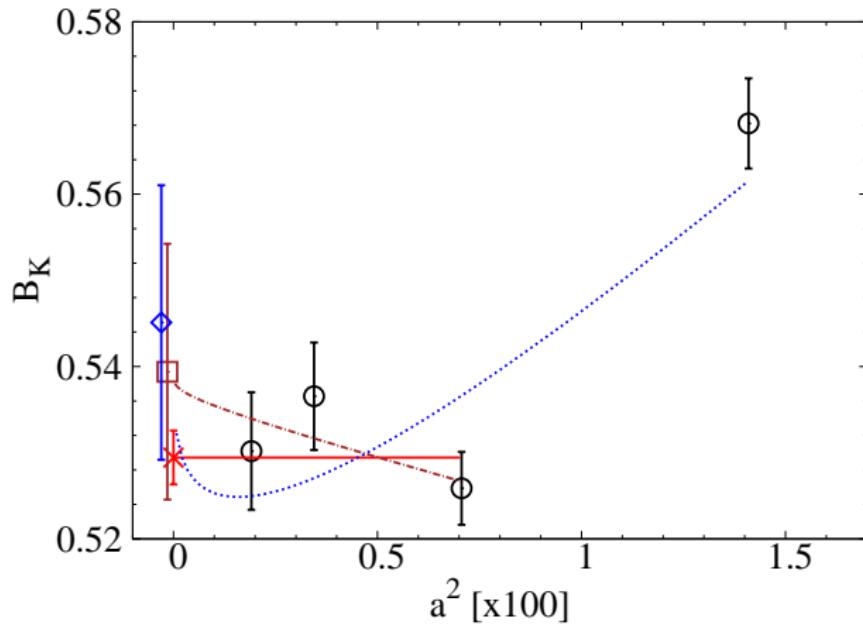
(a) X-fit



(b) Y-fit

- Fit type = SU(2) $S\chi$ PT, 4X3Y, NNNLO, Bayesian, FV

B_K vs. a^2 : Discretization Error



- Fit type = SU(2) SChPT; **const**, **3pt:g4-a2g2-a4**,
4pt:g4-a2g2-a4 fits

Error Budget of B_K [SU(2)-SChPT, 4X3Y, NNNLO]

cause	error (%)	memo
statistics	0.59	4X3Y-NNNLO-BAYES + const
matching factor	4.4	$\Delta B_K^{(2)}$ (U1)
discretization	1.9	diff. of const and constrained fits
X-fits	0.33	varying Bayesian priors (S1)
Y-fits	0.07	diff. of linear and quadratic (C3)
am_l extrap	1.5	diff. of (C3) and linear extrap
am_s extrap	1.3	diff. of (C3) and linear extrap
finite volume	0.5	diff. of $V = \infty$ and FV fits
r_1	0.14	r_1 error propagation (C3)
f_π	0.4	132 MeV vs. 124.4 MeV

Current Status of B_K (1)

- Lattice QCD (SWME):

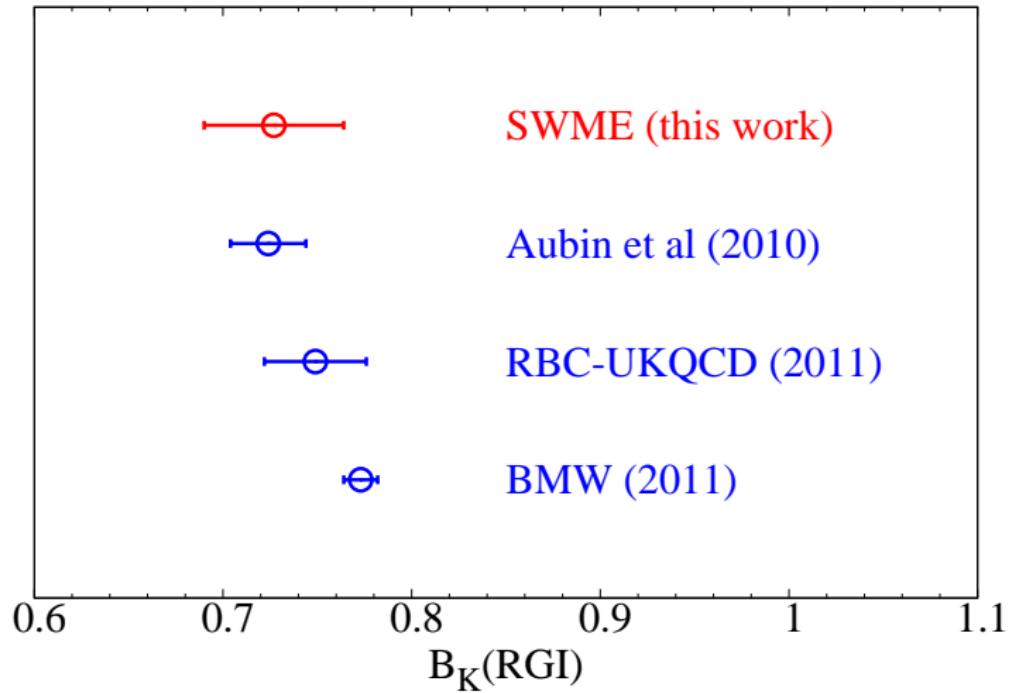
$$B_K(\text{RGI}) = \hat{B}_K = 0.727 \pm 0.004(\text{stat}) \pm 0.038(\text{sys})$$

- Experiments: (most updated version in 2012)

$$B_K(\text{RGI}) = \begin{cases} 1.01 \pm 0.11 & \text{for exclusive } V_{cb} \\ 0.824 \pm 0.060 & \text{for inclusive } V_{cb} \end{cases}$$

- Hence, we observe 2.6σ difference between the SM theory and experiments (exclusive V_{cb}) and 1.6σ difference (inclusive V_{cb}). Is this substantial?

Current Status of B_K (2)



Current Status of B_K (3)

- SWME (Stag):

$$\hat{B}_K = 0.727 \pm 0.004(\text{stat}) \pm 0.038(\text{sys})$$

- Laiho / Van de Water (DWF + Stag):

$$\hat{B}_K = 0.724 \pm 0.008(\text{stat}) \pm 0.028(\text{sys})$$

- RBC / UKQCD (DWF):

$$\hat{B}_K = 0.749 \pm 0.007(\text{stat}) \pm 0.026(\text{sys})$$

- BMW (Wilson):

$$\hat{B}_K = 0.773 \pm 0.008(\text{stat}) \pm 0.009(\text{sys})$$

Preliminary Theoretical Expectation for ε_K

- Inclusive V_{cb} :

$$\varepsilon_K = \begin{cases} 2.07 \pm 0.16 & (\text{Lat Avg}) \\ 1.95 \pm 0.19 & (\text{SWME}) \end{cases}$$

- Exclusive V_{cb} :

$$\varepsilon_K = \begin{cases} 1.66 \pm 0.21 & (\text{Lat Avg}) \\ 1.56 \pm 0.22 & (\text{SWME}) \end{cases}$$

- Experiment:

$$\varepsilon_K = 2.228 \pm 0.011 \quad (\text{PDG})$$

- We observe about 3.0σ tension in the exclusive V_{cb} channel of ε_K .

Preliminary check for CKM unitarity

- CKM unitarity:

$$[VV^\dagger]_{ct} = 0$$

- Exclusive V_{cb} : Lattice 2012 (Yong-Chull Jang)

$$[VV^\dagger]_{ct} = 0.00153(48) \quad (\text{preliminary})$$

- Inclusive V_{cb} : Lattice 2012 (Yong-Chull Jang)

$$[VV^\dagger]_{ct} = 0.00128(47) \sim 0.0084(47) \quad (\text{preliminary})$$

- We observe about 3.2σ tension in the exclusive V_{cb} channel of the CKM unitarity.

Preliminary error analysis with **inclusive V_{cb}**

- Lattice 2012: Yong-Chull Jang
- Lat. Avg.:

$$\left\{ \begin{array}{ll} \bar{\eta} & 27\% \\ \eta_3 & 19\% \\ m_c & 16\% \\ \dots & \dots \end{array} \right.$$

- SWME:

$$\left\{ \begin{array}{ll} \hat{B}_K & 34\% \\ \bar{\eta} & 18\% \\ \eta_3 & 12\% \\ m_c & 11\% \\ \dots & \dots \end{array} \right.$$

Preliminary error analysis with **exclusive** V_{cb}

- Lattice 2012: Yong-Chull Jang
- Lat. Avg.:

$$\left\{ \begin{array}{ll} V_{cb} & 58\% \\ \bar{\eta} & 11\% \\ \eta_3 & 9\% \\ m_c & 8\% \\ \dots & \dots \end{array} \right.$$

- SWME:

$$\left\{ \begin{array}{ll} V_{cb} & 49\% \\ \hat{B}_K & 18\% \\ \bar{\eta} & 9\% \\ \eta_3 & 8\% \\ m_c & 7\% \\ \dots & \dots \end{array} \right.$$

BSM corrections to B_K on the lattice

BSM operators for B_K

- BSM operators:

$$O_2 = \bar{s}_a(1 - \gamma_5)d_a\bar{s}_b(1 - \gamma_5)d_b$$

$$O_3 = \bar{s}_a(1 - \gamma_5)d_b\bar{s}_b(1 - \gamma_5)d_a$$

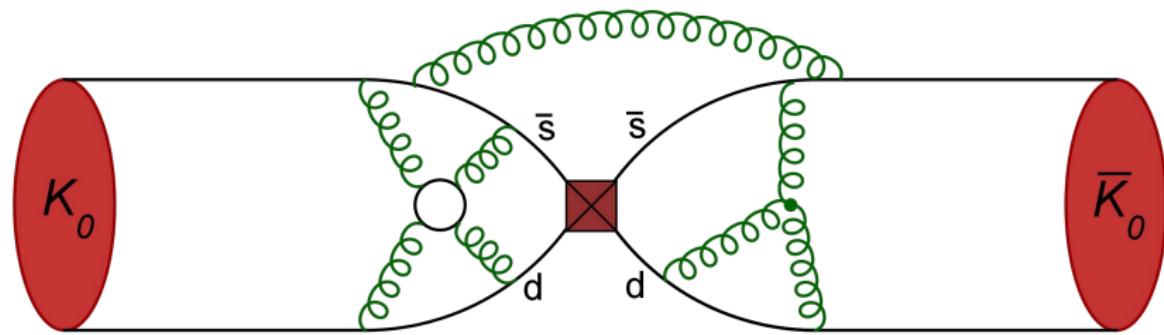
$$O_4 = \bar{s}_a(1 - \gamma_5)d_a\bar{s}_b(1 + \gamma_5)d_b$$

$$O_5 = \bar{s}_a(1 - \gamma_5)d_b\bar{s}_b(1 + \gamma_5)d_a$$

- B parameters:

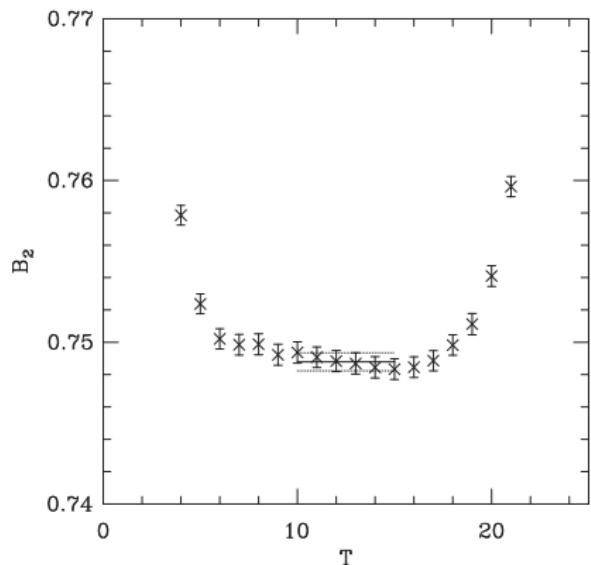
$$B_j(\mu) = \frac{\langle \bar{K}_0 | O_j(\mu) | K_0 \rangle}{N_j \langle \bar{K}_0 | P(\mu) | 0 \rangle \langle 0 | P(\mu) | K_0 \rangle}$$

What do we calculate on the lattice?

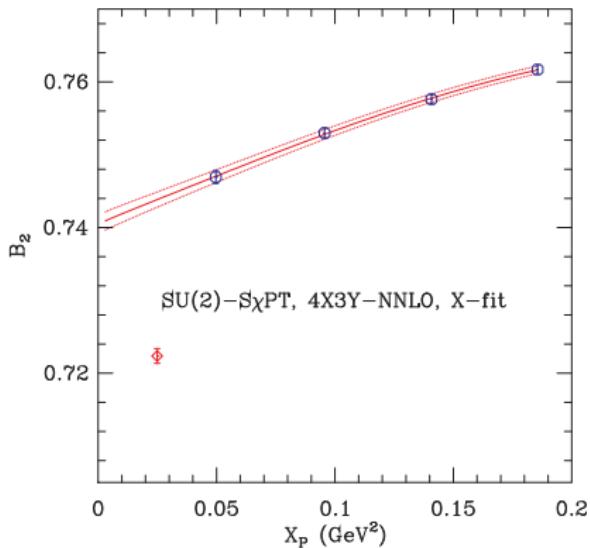


Data Analysis for BSM corrections to B_K

Data analysis for B_2 with $am_d = am_s = 0.025$ ($20^3 \times 64$)



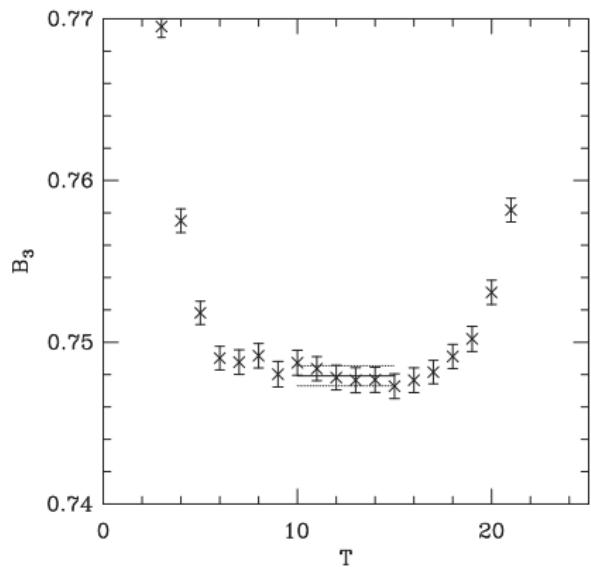
(c) raw data



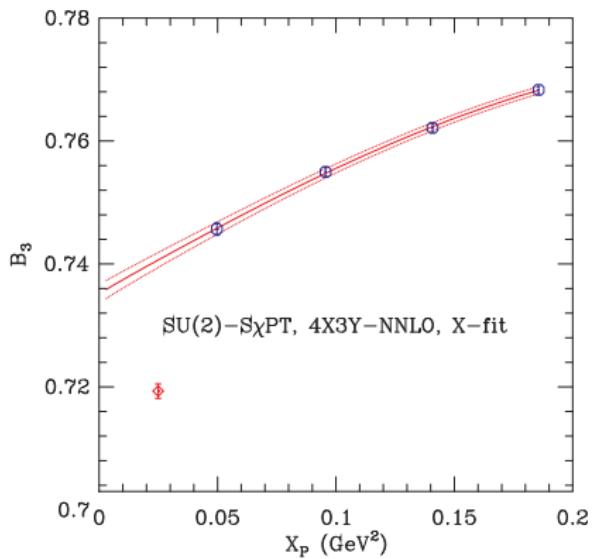
(d) X-fit

Preliminary !!!

Data analysis for B_3 with $am_d = am_s = 0.025$ ($20^3 \times 64$)



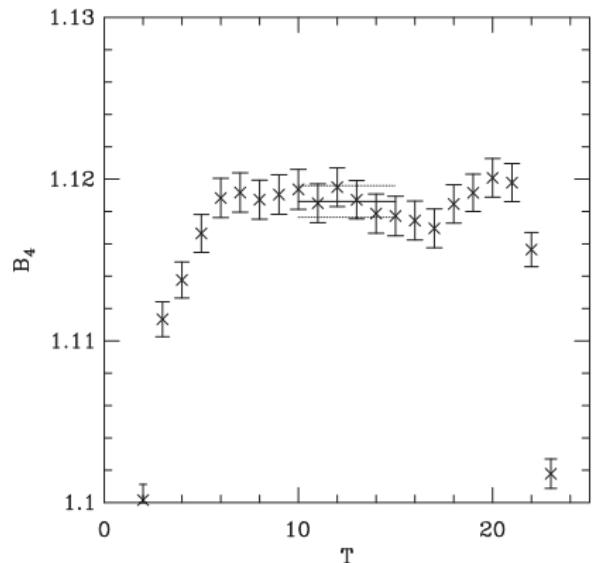
(e) raw data



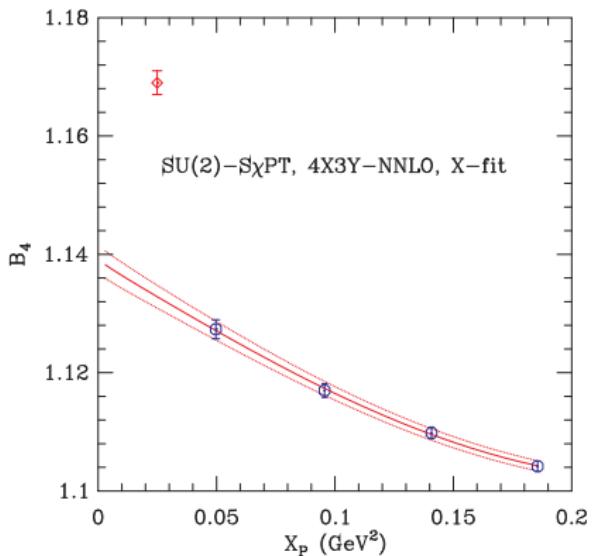
(f) X-fit

Preliminary !!!

Data analysis for B_4 with $am_d = am_s = 0.025$ ($20^3 \times 64$)



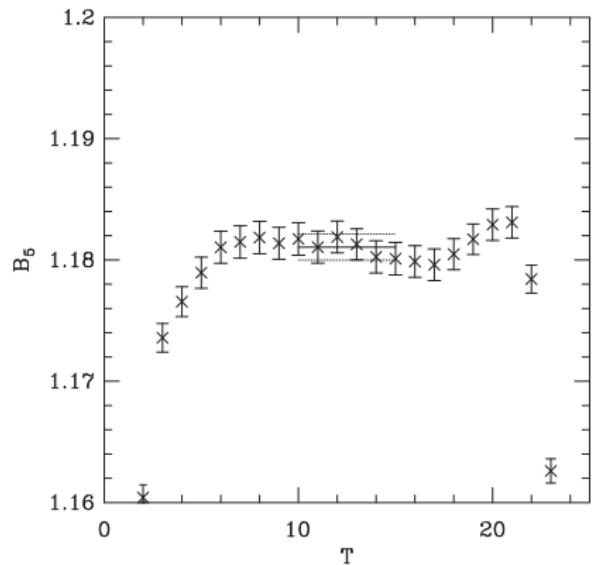
(g) raw data



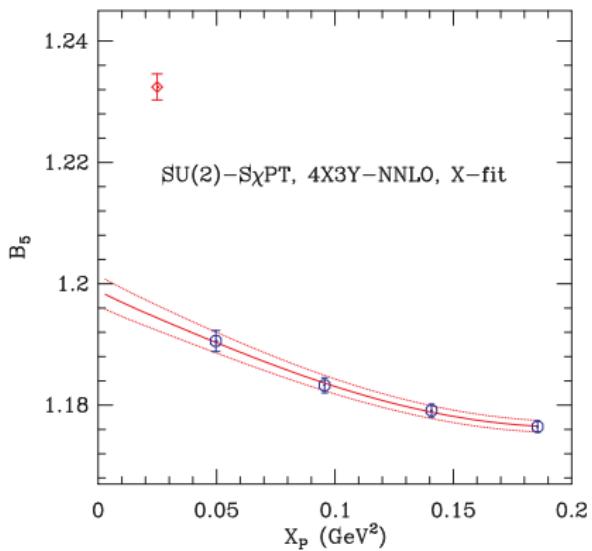
(h) X-fit

Preliminary !!!

Data analysis for B_5 with $am_d = am_s = 0.025$ ($20^3 \times 64$)



(i) raw data



(j) X-fit

Preliminary !!!

Current Status of BSM corrections to B_K

- We are in the middle of data analysis at the tree level. Hence the results are very **preliminary**.
- We plan to complete the first round data analysis at the one-loop level by Lattice 2012. We will present the preliminary results in Lattice 2012. [Dr. Hyung-Jin Kim]
- We plan to complete the second round data analysis using the NPR matching by the end of 2012.

Summary

- We plan to reduce the overall error of B_K below the 2% level. (at least for SWME). We have to reduce the overall error of exclusive V_{cb} below the 0.5% level.
- How?
Answer: NPR, or two-loop matching.
- Lattice 2012: Jangho Kim (NPR).
- Lattice 2012: Kwangwoo Kim (two-loop matching ???).

Future Theoretical Perspectives (1)

- Hansen & Sharpe (2012): multiple-channel scattering phase shift formula
- This directly applied to staggered $\pi - \pi$ scattering case: $N = 5$ channels.
- There is a remaining difficulty in the unitarity ansatz of the S-matrix.
- We will overcome this difficulty using the guidance of staggered chiral perturbation theory (SChPT).
- It could be very likely to calculate the $\pi - \pi$ scattering phase shift with the systematic errors under control in near future.
- This will make it possible to calculate the $K \rightarrow \pi\pi$ decay amplitude using staggered fermions.

Future Theoretical Perspectives (2)

- We have extended the SChPT calculation to non-Goldstone pion sectors for pion mass and decay constants f_π , f_K .
- We plan to extend the horizon to the mixed action case (HYP stag/asqtad or HYP/HISQ) in 2012.
- We plan to find the best channels to calculate the $\pi - \pi$ scattering phase shift using staggered fermions. The SChPT will be the main tool for this mission.